PHYSICAL REVIEW B **67**, 020503(R) (2003)

Domain wall superconductivity in ferromagnetic superconductors

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(Received 12 November 2002; published 13 January 2003)

On the basis of a phenomenological Ginzburg-Landau approach we investigate the problem of triplet order parameter nucleation in a ferromagnetic superconductor with a domain structure in an applied external magnetic field. The critical temperature of the superconductivity nucleation is shown to increase near the domain boundaries and to have a peculiar field dependence. Our results are also applied to a description of the superconductivity localized near the domain wall in S/F heterostructures.

DOI: 10.1103/PhysRevB.67.020503 PACS number(s): 74.25.Dw, 74.20.De, 74.20.Rp, 74.70.Tx

The problem of the coexistence of superconducting and magnetic orderings has been studied for several decades (see, e.g., Refs. 1, and 2 for review). One can distinguish two basic mechanisms responsible for the interaction of a superconducting order parameter with magnetic moments in the ferromagnetic state: (i) the electromagnetic mechanism (interaction of Cooper pairs with magnetic field induced by magnetic moments), which was first discussed by Ginzburg[3] in 1956, and (ii) the exchange interaction of magnetic moments with electrons in Cooper pairs. The revival of interest in the fundamental questions of magnetism and the coexistence of superconductivity has been stimulated by the recent discovery of ferromagnetic superconductors UGe_2 (Refs. 4,5) and URhGe (Ref. 6) where superconductivity appears in the presence of a large exchange field which obviously excludes singlet superconducting pairing. Note that in UGe_2 the superconductivity and ferromagnetism disappear simultaneously with a pressure increase. Such a behavior was explained in Refs. 7 and 8 in the framework of the theory of singlet superconductivity mediated by electron interaction via a localized spin. However, the subsequent discovery of superconductivity in URhGe below 0.3 K in ferromagnetic phases which have a Curie temperature $\Theta = 9.5$ K made the singlet scenario of superconductivity rather improbable. Therefore, we will assume a case of triplet ferromagnetic superconductivity.

The concrete structure of the unconventional order parameter in these systems is a subject of intensive discussions[9–12] and, thus, we can conclude that the problem of the choice of a reliable microscopic model for these compounds is still unresolved. Nevertheless, a number of questions important for experimentalists (in particular, the basic features of a magnetic field–temperature phase diagram) can be analyzed even in the absence of such a microscopic theory, if we start from the Ginzburg-Landau approach. The presence of domains is inherent to all ferromagnets of macroscopical sizes, and as the superconductivity appears in the ferromagnetic phase it is important to study the influence of the domains on the superconducting characteristics of ferromagnetic superconductors. In this paper we use phenomenological theory for an analysis of the effect of the domains on the critical temperature of superconducting nucleation.

For spin singlet superconducting pairing the conditions for the formation of a superconducting nucleus localized at the domain wall were studied in Ref. 13. Here we focus on the quite different case of triplet pairing which is not destroyed by the exchange field. Still the exchange interaction plays an important role even in this scenario: Cooper pairs with spin orientations parallel to the magnetization should be energetically more favorable than those with opposite spin orientations. As for the orbital effect (the electromagnetic mechanism), its influence on the superconducting ordering strongly depends on the specific domain structure inside the sample. We restrict ourselves to the most typical situation when the distance between domain walls is much less than the sample dimension in the wall plane. This choice allows us to consider the magnetic induction generated by the ferromagnetic moment to be almost uniform inside the domains.

In recent years hybrid S/F systems have attracted a growing interest due to their large potential for applications. In this connection note that, without taking account of the exchange interaction, the phase diagram of our system should be analogous to that of a thin-film structure consisting of a ferromagnetic insulator film (with a magnetization perpendicular to the plane and a thickness much larger than the distance between domain walls) and the superconducting film deposited on it. A similar situation can be obtained with a metallic ferromagnet when a superconducting film is evaporated on the buffer oxide layer in order to avoid a proximity effect. In both cases, with a decrease of the temperature the superconductivity must first appear just above the domain wall. Our case is somewhere complementary to the situation discussed in Ref. 14, where the S/F hybrid structure with in plane magnetization was been considered. For such a system the domain wall produces a local field in the S layer, which locally weakens the superconductivity.

At high temperatures the superconductivity inside the domains can be completely suppressed due to the orbital effect. Conversely, near the boundary the superconducting nucleus can be still energetically favorable due to a mechanism analogous to the one responsible for the existence of H_{c3} critical field for superconducting nucleus near the superconductor-insulator interface (see, for example, Ref. 15). Thus a change of the magnetization direction which occurs at a domain boundary is responsible for a partial decrease of the orbital effect which provides conditions for the formation of localized superconducting nuclei at domain walls at high temperatures (above the critical temperature inside the domains). Such a localized nucleus can appear only if we take account of the proximity effect, i.e., consider that Cooper pairs with a given spin orientation exist on both sides of the domain boundary. Obviously, the effect of the exchange field mentioned above should suppress the formation of these localized structures, since it produces a shift in the critical temperature for Cooper pairs with different spin orientations with respect to the magnetic moment. Such systems can reveal an interesting behavior in an external magnetic field. An external magnetic field applied to our sample (for simplicity, here we consider only a field applied parallel to the ferromagnetic moment) results in a partial compensation of magnetic induction in one of the domains. As a result, the critical temperature of the superconductor can depend nonmonotonically on the applied magnetic field. Both the critical temperature of the superconducting nucleation inside this domain and the critical temperature of the formation of localized superconductivity should increase up to an external field value equal to the magnetic induction induced by the ferromagnetic moment.

Let us choose the coordinate system with the z axis parallel to the a direction (magnetic easy axis), $x\| b$, $y\| c$ (see the notations for crystal axes of UGe_2 in Refs. 4, 5). We assume that within Ginzburg-Landau (GL) theory the superconducting state can be described by two components of the order parameter η_- and η_+, corresponding to Cooper pair spin orientations parallel and antiparallel to the z direction. These two components of the order parameter can be viewed, for instance, as corresponding to two different one-dimensional irreducible representations of the symmetry group of the crystal. Close to the nucleation temperature the fourth-order terms in the GL functional can be neglected, which allows us to introduce the GL free energy density in the form $F=F_-+F_+$, where

$$F_\pm = a_\pm |\eta_\pm|^2 + K_1|D_y\eta_\pm|^2 + K_1|D_z\eta_\pm|^2 + K_2|D_x\eta_\pm|^2, \tag{1}$$

$\mathbf{D}=\nabla-2\pi i\mathbf{A}/\phi_0$, $a_\pm=\alpha_\pm(T-T_{c0}\mp\Delta T_c^{ex}M_z/M)$, and the value ΔT_c^{ex} characterizes the shift of critical temperature due to the exchange mechanism (i.e., for a magnetization $\mathbf{M}$ chosen in the z direction $T_{c0}+\Delta T_c^{ex}$ is the transition temperature without orbital effect). Thus we take account of both mechanisms responsible for the interaction of the order parameter with in magnetization: (i) first, the orbital effect of the magnetic induction (electromagnetic mechanism), and (ii) second, the shift of the superconducting critical temperature due to the exchange interaction of the Cooper pair spin with the ferromagnetic moment. For an external magnetic field $\mathbf{H}$ applied in the z direction the total magnetic induction can be written as follows: $\mathbf{B}=\mathbf{H}+4\pi\mathbf{M}$. For a single magnetic domain we take the magnetization $\mathbf{M}$ in the form $\mathbf{M}=\mathbf{z}Mf(x)$, where $f(x)=\mathrm{sgn}(x)$ (we assume that the domain wall width is much less than the superconducting coherence length). Hereafter we choose the gauge in the form $\mathbf{A}=\mathbf{y}(4\pi M|x|+Hx)$. The linearized GL equations read

$$K_1(D_y^2+D_z^2)\eta_\pm + K_2D_x^2\eta_\pm - a_\pm\eta_\pm = 0. \tag{2}$$

The momenta along the z and y axes are conserved and one can search for the solution of Eq. (2) in the form $\eta_\pm=\exp(ik_zz+ik_yy)\Psi_\pm(x)$, where the function $\Psi_\pm$ should be found from a solution of the following one-dimensional problem:

$$-K_2\frac{\partial^2\Psi_\pm}{\partial x^2}+K_1k_z^2\Psi_\pm+K_1\times\left(k_y-\frac{2\pi}{\phi_0}(4\pi Mxf(x)+Hx)\right)^2\Psi_\pm\mp\alpha_\pm\Delta T_c^{ex}f(x)\Psi_\pm=\alpha_\pm(T_{c0}-T)\Psi_\pm. \tag{3}$$

Provided $\alpha_-=\alpha_+=\alpha$, the equation for Ψ_- can be obtained from the one for Ψ_+ if we change $x\to -x$ and $H\to -H$. Thus we can consider only the solution of the equation for Ψ_+ which has a nontrivial decaying solution for a set of discrete temperatures $T^+=T_n^+(k_y,k_z,H,M)$. Changing the sign of H, one can obtain a set of eigenvalues of the equation for the order parameter component Ψ_-: $T^-=T_n^+(k_y,k_z,-H,M)$. The highest eigenvalue $T_c=\max[T^+,T^-]$ corresponds to the critical temperature of the superconducting order parameter nucleation. It is obvious that T_c is maximal for the momentum $k_z=0$; hence hereafter we omit the term proportional to k_z^2 in Eq. (3). It is convenient to rewrite Eq. (3) in the dimensionless form

$$-\frac{\partial^2\Psi_+}{\partial t^2}+(tf(t)+ht-t_0)^2\Psi_+-\tau f(t)\Psi_+=E\Psi_+, \tag{4}$$

where $t=x/L$, $t_0=\phi_0k_y/(8\pi^2ML)$, $L^2=\sqrt{K_2/K_1}\phi_0/(8\pi^2M)$, $h=H/(4\pi M)$, $\tau=\Delta T_c^{ex}/\Delta T_c^{orb}$, $E=(T_{c0}-T)/\Delta T_c^{orb}$, and the value $\Delta T_c^{orb}=8\pi^2M\sqrt{K_1K_2}/(\alpha\phi_0)$ characterizes the shift of critical temperature due to the orbital mechanism. For the case $|t_0|\to\infty$ a superconducting nucleus will appear either in the left or right half-space far from the domain boundary. In this limit the lowest eigenvalue $E=\min[-\tau+|1+h|,\tau+|1-h|]$ of Eq. (4) and, hence, the critical temperature are not disturbed by the presence of the domain boundary. This critical temperature T_c^{bulk} corresponds to the order parameter nucleation and to the formation of a vortex state inside the domains. Conversely, for finite t_0 values the superconducting nuclei in different half-spaces cannot be considered separately due to the proximity effect. Provided the lowest energy level in the resulting potential well in Eq. (4) is minimal for a certain finite t_0 coordinate, we obtain a superconducting nucleus localized at the domain boundary for temperatures above T_c^{bulk}. The mechanism resulting in the appearance of such localized nucleus is analogous to the one responsible for the existence of the surface superconductivity at the superconductor/insulator boundary for magnetic fields $H_{c2}<H<H_{c3}$. Indeed, for $\tau=0$ and h

$=0$ the potential well $V(x)$ in the Schrödinger equation [Eq. (4)] is symmetric $[V(x)=V(-x)]$ and the eigenvalue problem [Eq. (4)])can be considered only for the right half-space $t>0$ with the boundary condition $\Psi'_+(t=0)=0$. For this particular case the energy minimum corresponds to $t_0^2=E_{min}=0.59010$.[15] An increase in the h and τ values will obviously result in an increasing asymmetry of the well $V(x)$, and, thus, in the suppression of localized superconductivity. Equation (4) can be solved exactly in terms of Weber functions (see Refs. 15,16):

$$\Psi_+=AW\left(\sqrt{1+h}t-\frac{t_0}{\sqrt{1+h}},\frac{E+\tau}{1+h}\right),\quad t>0, \tag{5}$$

$$\Psi_+=BW\left(-\sqrt{1-h}t-\frac{t_0}{\sqrt{1-h}},\frac{E-\tau}{1-h}\right),\quad t<0. \tag{6}$$

Here A and B are constants, and the Weber function $W(s,\varepsilon)$ is the solution of the equation

$$-\frac{\partial^2 W}{\partial s^2}+s^2W=\varepsilon W, \tag{7}$$

with the boundary condition $W(s\to+\infty,\varepsilon)\to 0$. Matching these solutions at $t=0$ we obtain:

$$\frac{\sqrt{1+h}W_s'\left(-\frac{t_0}{\sqrt{1+h}},\frac{E+\tau}{1+h}\right)}{W\left(-\frac{t_0}{\sqrt{1+h}},\frac{E+\tau}{1+h}\right)}=-\frac{\sqrt{1-h}W_s'\left(-\frac{t_0}{\sqrt{1-h}},\frac{E-\tau}{1-h}\right)}{W\left(-\frac{t_0}{\sqrt{1-h}},\frac{E-\tau}{1-h}\right)}. \tag{8}$$

This equation can be solved numerically, which allows us to obtain the function $E(t_0,h,\tau)$. The resulting dependence of the critical temperature of superconductivity nucleation on parameters h and τ is shown in Fig. 1. One can see that both the external field (h) and the exchange interaction (τ) suppress the localized superconducting nuclei, and the superconductivity localized on the domain wall exists only at a relatively weak applied field. Note that for the case $\tau=0$ [see Fig. 1(a)] we obtain the phase diagram of superconducting film on a ferromagnet with a domain wall.

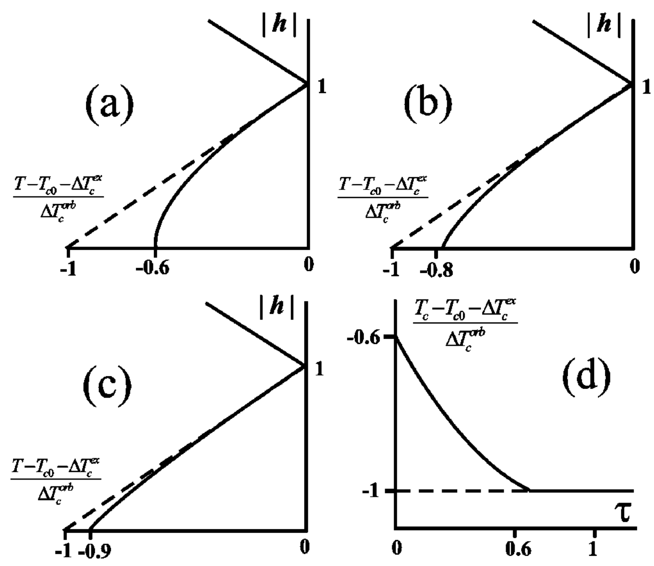

FIG. 1. (a)–(c) Critical external field (h) of the superconductivity nucleation as a function of the temperature: (a) for the parameter $\tau=0$, (b) for the parameter $\tau=0.2$, and (c) for the parameter $\tau=0.4$. (d) Critical temperature of the superconductivity nucleation as a function of the exchange interaction (the parameter τ) for $h=0$. The solid (dashed) lines correspond to superconducting nucleation at the domain boundaries (inside the domains).

Finally we would like to discuss some estimates of the physical parameters for the systems where the nucleation of superconductivity at domain boundaries could be observable. Taking a magnetization of $4\pi M\sim 2$ kOe and a slope of the temperature dependence of the upper critical field of $dH_{c2}/dT\sim 40$ kOe/K for the case of UGe_2,[4,5] we obtain $\Delta T_c^{orb}\sim 0.05$ K. Thus, at zero external magnetic field the increase in the temperature of superconductivity nucleation at domain boundaries may be of the order of $\delta T_c\sim 0.02$ K ~ 0.05 T_c. This estimate gives us a quite measurable temperature interval. The parameters for URhGe (Ref. 6) are of same order of magnitude. For S/F heterostructures we can take, for example, the parameters of Nb ($T_c\sim 9$ K and $dH_{c2}/dT\sim 0.5$ kOe/K) and typical values of magnetization for ferromagnetic insulators of $4\pi M\sim 1-10$ kOe. The resulting increase in the critical temperature above a domain wall is quite strong: $\delta T_c\sim 1$–3 K. The thickness of a superconducting film must be much smaller than the distance between domains, and ideal conditions correspond to the thickness of the order of several coherence lengths. So we conclude that the effects discussed above may be easily observed and could be quite important.

To summarize, we investigated the conditions for the existence of localized triplet superconductivity at domain boundaries in ferromagnetic superconductors. The appearance of these localized superconducting nuclei should result in a broadening of the superconducting transition probed by the resistivity measurements. In fact, the beginning of the resistivity decrease with the temperature decrease would correspond to the domain-wall superconductivity, while its complete disappearance would signal the bulk superconductivity. An external magnetic field would shrink the region of domain-wall superconductivity. The experimental observation of the (H, T) diagram discussed above for UGe_2 and URhGe could provide arguments in favor of unconventional pairing in these compounds, and permit one to determine the important parameter τ, describing the ratio of exchange and orbital effects. Note, in conclusion, that the existence of localized superconducting channels near the domain walls in S/F heterostructures can provide the interesting possibility to realize a switching behavior, provided we can move the ferromagnetic domain wall. The superconducting channel in this case should follow the motion of the domain wall, which provides the possibility to control the conductance between certain static leads.

We thank C. Meyers, V. P. Mineev, J.-P. Brison, and J. Flouquet for useful discussions. This work was supported, in part, by the Program of the Russian Academy of Sciences ''Quantum Macrophysics,'' ESF ''Vortex'' Program and Theoretical Institute at the Argonne National Laboratory. A.S.M. is grateful to the members of the Condensed Matter Theory Group at University Bordeaux I, for their kind hospitality.

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