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Metastable and nonuniform states in 2D orthorhombic dipole system

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Abstract

The energy of a system of 3D dipoles on a 2D orthorhombic lattice is investigated as a function of the rhombic angle. The possibility of the metastable ferromagnetic state above the main antiferromagnetic state is shown. The mechanisms of magnetization reversal are analyzed. Nonuniform structures with the long range order of the ferromagnetic type (vortices and domain walls) on bounded lattices are considered. The conditions when the upper terms of the long wave expansion of the Fourier images of the dipole sums plays a significant role are shown. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

For the first time a possibility of existence of the long range order (LRO) in 3D systems with a dipole–dipole interaction only was shown by Lattinger and Tisza [1,2]. This prediction was experimentally confirmed for the magnetic properties of the rare-earth combinations which were named 'dipole magnetics' (see e.g., Ref. [3]). The temperature of the phase transition is not higher than 1 K in these combinations. The reason is that the exchange interaction is absent and the dipole interaction between the magnetic moments of the rare-earth atoms is weak. Now there is a possibility of an experimental investigation of 2D dipole magnetics. Such a situation may be realized in a system of single-domain ferromagnetic particles on a nonmagnetic dielectric substrate [4–9]. Nowadays the technology of preparing an ordered system of single-domain ferromagnetic particles is rapidly progressing due to the possibility of using such systems for super dense record and storage of information [10]. It should be pointed out that the expected phase transition temperature for these systems is essentially higher than in the 'classical' 3D dipole magnetics [11,12]. An increase of the critical temperature in the system of single-domain particles is caused by a large value of their magnetic

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moments, which is proportional to the number of atoms in the particle.

Thus, a theoretical investigation of a system of dipoles on a 2D lattice is of significant interest. The main state of such systems was studied in [11,13–15]. For the orthorhombic lattices it was shown that the main state has the LRO of the ferromagnetic (FM) type, if the rhombic angle is less than some critical value ($\varphi_c \approx 76°$ [11]). For greater values of the rhombic angle $\varphi > \varphi_c$ there is the main state with LRO of the antiferromagnetic (AFM) type. It is necessary to make two remarks here. First, the phases with FM and AFM type of LRO have different symmetries and the phase transition between them must be of the first type. It means that the phase with LRO of the FM type may be metastable on the lattices with $\varphi > \varphi_c$. Second, the FM phase in the bounded systems is nonuniform and must divide in the domains [16].

Our work is devoted to the investigation of metastable and nonuniform states of a system of magnetic dipoles with FM type of LRO on a 2D orthorhombic lattice. The used model is formulated and the procedure of transformation of the Fourier images of 2D dipole sums into rapidly convergent sums of the McDonald functions (suggested in Ref. [17]) is described in Section 2. This procedure allows to obtain an analytical expression for the energy of the system in the long wave approximation and make numerical calculation much easier. In Section 3 the stability of the phase with the FM type of LRO is investigated in a system with a large value of the rhombic angle (with the AFM type of the main state). It was found out that it is metastable on all lattices with $\varphi < 90°$. Besides, the stability in the external field is analyzed for all rhombic angles and it is shown that the magnetization reversal of the system takes place either by coherent rotation of the magnetic moments in the plane of the system in the case of large rhombic angles, or by antiferromagnetic fanning, if $\varphi > 44°$. Section 4 is devoted to the investigation of the nonuniform states of a dipole system on bounded lattices (vortices of domain walls). The situations when the highest terms in the long wave expansion of the Fourier images of the dipole sums which are formally similar to exchange interaction but have a pure dipole nature play a significant role are analyzed. The analytical expression for magnetization distribution in the vortex and domain wall in the external field near saturation is obtained. It is shown that in the weak field, when the domain wall is getting thin, the process of the magnetization reversal is determined by the wall pinning on the lattice. All the results are summarized in Section 5.

2. Calculation of the dipole tensor components for 2D orthorhombic lattices

The energy of a system of 3D dipoles with the magnetic moments $\boldsymbol{M}(\boldsymbol{r}_i)$ on a 2D lattice is defined as

$$E = \tfrac{1}{2}\sum_{i \neq j} M_\alpha(\boldsymbol{r}_i) D^{\alpha\beta}(\boldsymbol{r}_{i,j}) M_\beta(\boldsymbol{r}_j), \quad \alpha, \beta = x, y, z; \tag{1}$$

$$D^{\alpha\beta}(\boldsymbol{r}_{i,j}) = \frac{\delta_{\alpha\beta}}{r_{i,j}^3} - \frac{3r_{i,j}^\alpha r_{i,j}^\beta}{r_{i,j}^5}, \quad \boldsymbol{r}_{i,j} = \boldsymbol{r}_i - \boldsymbol{r}_j, \tag{2}$$

where i, j number the lattice site. Here and below, the repeated indices denote summation. $D^{xz} = D^{yz} = 0$ for the 2D system. The absolute value of the magnetic moment of the dipole is M_0. We consider the orthorhombic lattices which have the rhombic angle φ and its lattice translation vector $a_0 = 1$ (Fig. 1). Then the diagonal sizes are $a = 2\sin(\varphi/2)$ and $b = 2\cos(\varphi/2)$. The energy of the system in the Fourier representation is

$$E = \tfrac{1}{2}N\sum_{\boldsymbol{q}} M_\alpha(\boldsymbol{q}) D^{\alpha\beta}(\boldsymbol{q}) M_\beta(-\boldsymbol{q}), \tag{3}$$

$$D^{\alpha\beta}(\boldsymbol{q}) = \sum_{i \neq j}\left(\frac{\delta_{\alpha\beta}}{|\boldsymbol{r}_{i,j}|^3} - \frac{3r_\alpha r_\beta}{|\boldsymbol{r}_{i,j}|^5}\right)\exp(-\mathrm{i}\boldsymbol{q}\boldsymbol{r}_{i,j}), \tag{4}$$

$$M_\alpha(\boldsymbol{q}) = N^{-1}\sum_{\boldsymbol{r}} M_\alpha(\boldsymbol{r})\exp(-\mathrm{i}\boldsymbol{q}\boldsymbol{r}), \tag{5}$$

where N is the number of lattice sites.

To describe the system, it is necessary to find the Fourier representation of the components of the dipole tensor (4). There are several methods to do this. One is the Ewald method which is used, for example, in Ref. [18] for calculation of the dipole sums in 3D magnetics. In our work we have used the 'chain' method suggested by Van der Hoff and Benson [17] and presented in detail in Ref. [19] for

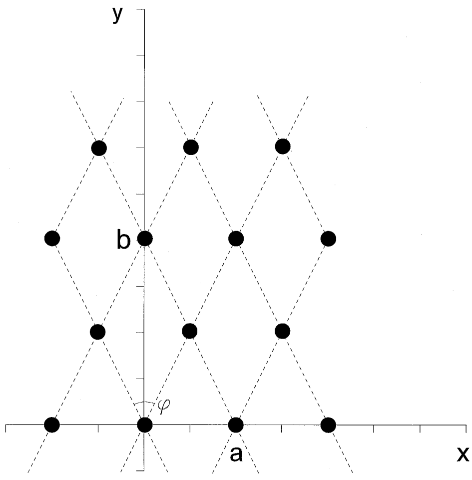

Fig. 1. The lattice under study. The coordinate axes are directed along the rhombus diagonals, a and b are the longitudes of the diagonals, φ is the rhombic angle.

definition of the dipole tensor components if $\boldsymbol{q}=0$ in the 3D case. The main idea of the method is to pick out the chains of the dipoles on the lattice so that the distance between the nearest sites in them will be minimal. Formally, as will be shown later, the double sums in Eq. (4) transform to the rapidly convergent sums of the McDonald's functions. The effectiveness of the method (the number of the series terms needed to be taken into account) depends on the choice of the chains or, in other words, on the sequence of the summation in Eq. (4). Briefly, the method can be exemplified by the calculation of $D^{zz}(\boldsymbol{q})$

$$D^{zz}(\boldsymbol{q})=\sum_{i\neq j}\frac{\exp(-\mathrm{i}\boldsymbol{q}\boldsymbol{r}_{i,j})}{|\boldsymbol{r}_{i,j}|^3}. \tag{6}$$

Let us choose the chains along the short diagonal of the rhombus, i.e., along the x axis (Fig. 1). Then Eq. (6) yields

$$D^{zz}(\boldsymbol{q})=\sum_{m\neq 0}\frac{\exp(-\mathrm{i}q_x am)}{(am)^3}+\sum_{n\neq 0}\exp\left(-\frac{\mathrm{i}q_y bn}{2}\right)L_n, \tag{7}$$

$$L_n=\sum_m\frac{\exp\left(-\mathrm{i}q_x\left(\frac{an}{2}-am\right)\right)}{\left(\left(\frac{an}{2}-am\right)^2+\left(\frac{bn}{2}\right)^2\right)^{3/2}}. \tag{8}$$

In Eq. (7) the first term corresponds to the isolated chains and the second one corresponds to the interaction between them. By use of the representation of the gamma function

$$\Gamma(z)=\mu^z\int_0^\infty t^{z-1}\exp(-\mu t)\,\mathrm{d}t,$$

for L_n we can have

$$L_n=\frac{2}{\sqrt{\pi}}\int_0^\infty t^{1/2}\exp\left(-\left(\frac{bn}{2}\right)^2 t\right)\times\sum_m\exp\left(-\mathrm{i}q_x\left(\frac{an}{2}-am\right)\right)\times\exp\left(-\left(\frac{an}{2}-am\right)^2 t\right)\mathrm{d}t. \tag{9}$$

The sum under the integral is evaluated by the Poisson summation formula and using the integral representation of the modified Bessel function of the second kind of order v (McDonald's function) [20]:

$$K_v(2\sqrt{\beta\gamma})=\frac{1}{2}\left(\frac{\gamma}{\beta}\right)^{v/2}\int_0^\infty t^{v-1}\exp(-\beta/t-\gamma t)\,\mathrm{d}t.$$

For $D^{zz}(\boldsymbol{q})$ one can obtain

$$D^{zz}(q_x,q_y)=2\sum_{m=1}\frac{\cos(q_x am)}{(am)^3}+\frac{8}{a}\sum_{n=1}^{\infty}\cos(q_y bn/2)\left|\frac{q_x}{bn}\right|K_1\left(\left|\frac{q_x bn}{2}\right|\right)+\frac{8}{a}\sum_{n=1}^{\infty}\cos(q_y bn/2)\times\sum_{h\neq 0}(-1)^{nh}\left|\frac{q_x+2\pi h/a}{bn}\right|\times K_1\left(\left|\frac{bn}{2}\left(q_x+\frac{2\pi h}{a}\right)\right|\right). \tag{10}$$

Expression (10) for $D^{zz}(q_x,q_y)$ is accurate. The first term is calculated analytically. The second and the third ones are rapidly convergent sums excluding the points $q_x = 2\pi l/a, l = 0, \pm 1 \ldots$, but in these points the sums should be calculated as the first term. The simple analytical expressions can be obtained from Eq. (10) in two important cases, which would be used in Section 4 while analyzing nonuniform states. In the case when the magnetic moment depends only on one coordinate ($q_x = 0$):

$$D^{zz}(q_y) = \frac{2\zeta(3)}{a^3} + \frac{8\pi^2}{3ab^2} - \frac{4\pi q_y}{ab} + \frac{q_y^2}{a} - \frac{32\pi^2}{a^2 b}\cos\left(\frac{q_y b}{2}\right) K_1\left(\frac{\pi b}{a}\right). \quad (11)$$

Here $\zeta(3) \approx 1.20205$ is the Rieman's zeta-function. In this expression only the first term of the third sum is taken into account, as the McDonald's function $K_1(z)$ exponentially decreases with an increase in its argument. The expressions for the other components of the dipole tensor can be obtained by the same method.

If the magnetic moment is a slow function of the coordinates, the long wave approximation of the dipole tensor can be obtained. The first three terms in the expansion in terms of q are

$$D^{zz}(\boldsymbol{q}) = C_0^{zz} - \frac{4\pi}{ab}|\boldsymbol{q}| + B_x^{zz} q_x^2 + B_y^{zz} q_y^2, \quad (12)$$

where

$$C_0^{zz} = \frac{2\zeta(3)}{a^3} + \frac{8\pi^2}{3a^2 b} + \frac{32\pi}{ab^2}\sum_{n=1}\sum_{h=1}(-1)^{nh}\frac{h}{n}K_1\left(\pi\frac{a}{b}hn\right), \quad (13)$$

$$B_x^{zz} = \frac{1}{b}\left(1 - 4\sum_{n=1}\sum_{h=1}(-1)^{nh}\pi\frac{a}{b}hnK_1\left(\pi\frac{a}{b}hn\right)\right), \quad (14)$$

or

$$B_x^{zz} = \frac{1}{a}\left(\ln\left(\frac{8\pi a}{b}\right) - 1 + C - 4\sum_{n=1}\sum_{h=1}(-1)^{nh}\left(K_0\left(\pi\frac{b}{a}hn\right) - \pi\frac{b}{a}hnK_1\left(\pi\frac{b}{a}hn\right)\right)\right). \quad (15)$$

$C \approx 0.577215$ is the Euler constant. B_y^{zz} comes out from B_x^{zz}, if we change $a \leftrightarrow b$. The choice of the form for B_y^{zz} and B_x^{zz} from the two represented is defined by the ratio of the rhombus diagonals a, b. The best form ensures the most rapid convergence of the series. The expansions in the terms of q for the other components of the dipole tensor have the same form. Their expressions coincide in specific cases of the quadratic and triangular lattices with those represented in Ref. [11].

In the obtained series $C_0^{\alpha\alpha}$ constants define the magnetization direction of the dipole system in the uniform state, i.e., corresponds to anisotropy; the terms of the first order correspond to the magnetostatic terms in the continuum approximation; and, finally, the terms $\sim q^2$ are similar to the short-range exchange interaction, but their presence is caused by a discrete type of the system. It will be further referred as 'pseudoexchange'.

3. Metastable states of the dipole system with the ferromagnetic type of the long range order. The instability and magnetization reversal

In this section, we find the stable region of the phase with the FM type of LRO of the dipole system on the orthorhombic lattice. The ground state of the system was analyzed in detail in Ref. [11] and it was shown that there were our ground states depending on the rhombic angle. If the angle is small ($\varphi \leqslant \pi/3$), the dipoles are ordered ferromagnetically and the magnetic moment of the system is oriented along the short diagonal of the rhombus (the DF phase). In the range of $\pi/3 \leqslant \varphi \leqslant \varphi_c$ ($\varphi_c \approx 76°$), the DF_1 ground state existed. The dipoles are oriented ferromagnetically also, but the magnetic moment is directed along the long diagonal of the rhombus. If $\varphi = \pi/3$, the energy of the system is isotropic about the uniform rotation of the magnetization in the plane of the system, and the DF and DF_1 states have equal energy. If $\varphi_c \leqslant \varphi < \pi/2$, the ground state acquires antiferromagnetic character (AF). In this state the chains with the equally directed dipoles are stretched along the rhombus side, and the dipoles in the nearest chains are oriented in the opposite directions along chains. If $\varphi = \pi/2$ the microvortex state is realized [11]. So, if the rhombic angle is greater than 76°, the state with the FM type of LRO is not the main state. In spite of this fact, it can be realized as the DF_1 state is stable in that case also. Actually, the AF and DF_1 states have the symmetries which belong to different groups, and no one is the subgroup of the other. Therefore, the phase transition $DF_1 \to AF$ is of the first type and the DF_1 state can be metastable. To define the region of metastability, it is necessary to check the stability of DF_1 state about all possible small perturbations.

According to Eq. (3), the energy of the system in the DF_1 state is

$$E_0 = \tfrac{1}{2} N D^{yy}(0) M_0^2. \tag{16}$$

Let us consider a small perturbation in the form of

$$\boldsymbol{M}(\boldsymbol{r}) = M_0 \boldsymbol{y}_0 + \boldsymbol{\xi}(\boldsymbol{r}), \tag{17}$$

where $\boldsymbol{\xi}$ is small. Due to the constancy of the modulus of the magnetization vector and the infinitesimal nature of ξ, it is

$$\xi_y(\boldsymbol{r}) \approx -\frac{1}{2M_0}|\boldsymbol{\xi}|^2 \approx -\frac{1}{2M_0}\xi_\perp^2 = -\frac{1}{2M_0}(\xi_x^2 + \xi_y^2). \tag{18}$$

Keeping quadratic terms of ξ in the expression (3) for E, the change in energy is obtained:

$$E - E_0 = \tfrac{1}{2} N \sum_{\boldsymbol{q}} ((D^{xx}(\boldsymbol{q}) - D^{yy}(0))\xi_x(\boldsymbol{q})\xi_x(-\boldsymbol{q}) + (D^{zz}(\boldsymbol{q}) - D^{yy}(0))\xi_z(\boldsymbol{q})\xi_z(-\boldsymbol{q})). \tag{19}$$

Obviously, the system will be stable in the DF_1 state, if for all possible $\boldsymbol{q}$ the following conditions are fulfilled:

$$D^{xx}(\boldsymbol{q}) - D^{yy}(0) > 0, \tag{20}$$

$$D^{zz}(\boldsymbol{q}) - D^{yy}(0) > 0. \tag{21}$$

These conditions are fulfilled, if the minimal values of $D^{xx}(\boldsymbol{q})$ and $D^{zz}(\boldsymbol{q})$ are greater than $D^{yy}(0)$. The dipole sums were summed numerically with the help of the formulas from Section 2 and it was obtained that the minimal value of $D^{zz}(\boldsymbol{q})$ lies on the boundary of the Brillouin zone with $q_x = \pi/a \cos\varphi$, $q_y = 0$; the one of $D^{xx}(\boldsymbol{q})$ is in the center. In the whole range $\pi/3 < \varphi < \pi/2$ the conditions of stability are fulfilled. If $\varphi = \pi/3$ and $\varphi = \pi/2$, the DF and DF_1 states have equal energies. If $\varphi < \pi/3$, the DF_1 state becomes unstable about the perturbations which transforms it into the DF state. If $\varphi = \pi/2$, the state with the FM type of LRO is degenerated about the rotation of magnetization in the plane of the system ($D^{xx}(0) = D^{yy}(0)$) and for the stability it is necessary that the condition (21) be fulfilled for the arbitrary choice of the axes $0x$ an $0y$. Even if it is fulfilled for the DF_1 state, it is not fulfilled for the state with the FM type of LRO with the direction of magnetization along the side of the square. In this case $D^{zz}(\boldsymbol{q}) - D^{yy}(0) < 0$ ([21], Fig. 1) and DF_1 is unstable about its transformation in the AF state which has an energy equal to that of the microvortex state of the square lattice [11].

So the state with the FM type of LRO is stable for all the possible rhombic angles excluding the square lattice. If $\varphi \leqslant \varphi_c \approx 76°$, it corresponds to the ground state, in the range $\varphi_c \leqslant \varphi < \pi/2$ it is metastable.

Let us consider the question about stability of a state with the FM type of LRO in an external magnetic field and, accordingly, about the mode of the magnetization reversal. In the presence of the magnetic field H directed against magnetization the conditions of the stability of the DF_1 state take the form:

$$D^{xx}(0) - D^{yy}(0) - \frac{H}{M_0} > 0, \tag{22}$$

$$D^{zz}(\boldsymbol{q}_{af}) - D^{yy}(0) - \frac{H}{M_0} > 0. \tag{23}$$

While writing these conditions, we already use the minimal values of $D^{xx}(\boldsymbol{q}) = D^{xx}(0)$ and $D^{zz}(\boldsymbol{q}) = D^{zz}(\boldsymbol{q}_{af})$; $\boldsymbol{q}_{af}$ is the wave vector on the boundary of the Brillouin zone, where $D^{zz}(\boldsymbol{q})$ has its minimum. To define the type of the instability, it is necessary to compare the values of $D^{xx}(0)$ and $D^{zz}(\boldsymbol{q}_{af})$. The minimal one defines the mode of the magnetization reversal. The dependence of the critical fields of the magnetization reversal versus rhombic angle is represented in Fig. 2. If the rhombic angle is large enough ($\varphi > \varphi_{c1}$, where $\varphi_{c1} \approx 43.95°$), the anisotropy connected with the axis lying in the plane of the system is weak and the

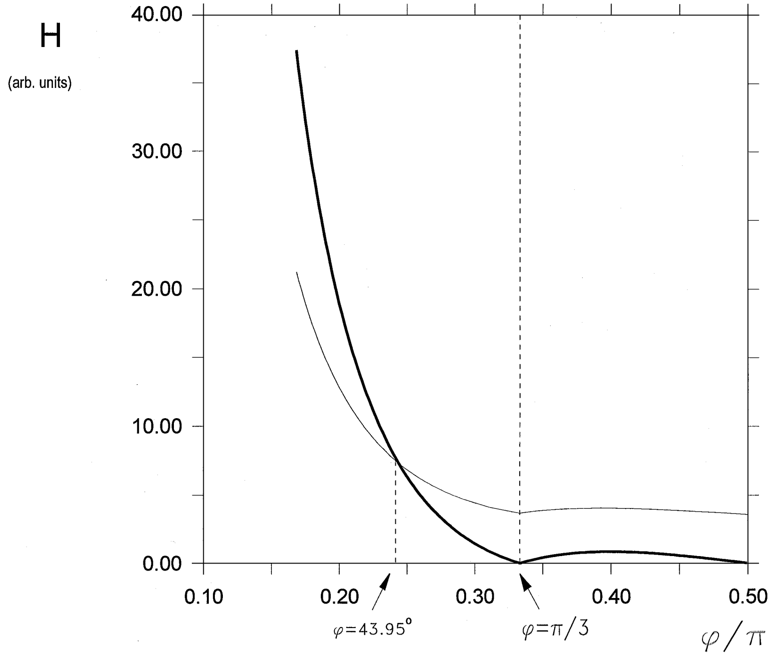

Fig. 2. The dependence of the reversal field on the rhombic lattice. The thin line corresponds to the critical field of antiferromagnetic rotation, the thick one to the field, when the coherent rotation of the magnetic moments takes place in the plane of the system. If $\varphi < \pi/3$ these dependencies are presented for the phase DF, if $\pi/3 < \varphi < \pi/2$ for the phase DF_1.

magnetization reversal takes place by coherent rotation of the magnetic moments in the plane of the system. With a decrease of the rhombic angle this anisotropy ($C_0^{xx} - C_0^{yy}$) increases, and the magnetization reversal takes place by antiferromagnetic fanning of the dipoles through the $0z$ axis in the weaker fields.

4. Vortices and domain walls in the dipole system with a ferromagnetic type of the long range order

The uniform state of a dipole system with the FM type of LRO cannot be the ground state on a bounded lattice, due to the large energy of the demagnetizing fields in this case. To reduce this energy, the dipole system will tend to be in the configuration without magnetic charges. It is well known that the usual exchange magnetics divides in domains and the width of the domain wall depends on the ratio between the exchange integral and the constant of anisotropy. In the case of an easy plane ferromagnet, creation of a vortex structure is possible [22]. A similar situation must be realized in the dipole system with the FM type of LRO. As has been shown in Section 2, the energy of the dipole system contains the terms similar to anisotropy and exchange which have, nevertheless, a magnetostatic nature. From the considerations of the dimensions it is clear that in the dipole system the domain wall width is comparable with the size of the unit cell (see also Ref. [23]). The situation significantly changes in an external magnetic field directed perpendicularly to the easy axis. In this section the analytic expressions are obtained for the domain wall and vortex in the external field for a 2D dipole system.

Let us consider the sample of a triangular lattice, which does not have the anisotropy axis in the plane. The numerical analysis for the 2D dipoles in such a sample shows that the ground state has a vortex structure [16]. We shall obtain analytical expressions for a configuration of 3D dipoles in an external magnetic field perpendicular to the sample plane. The long-wave asymptotics of the dipole tensor for the triangular lattice may be represented as [11]

$$D^{xx}(q) = D_{\perp}(q) + (D_{\parallel}(q) - D_{\perp}(q))\frac{q_x^2}{q^2}, \tag{24}$$

$$D^{yy}(q) = D_{\perp}(q) + (D_{\parallel}(q) - D_{\perp}(q))\frac{q_y^2}{q^2}, \tag{25}$$

$$D^{xy}(q) = (D_{\parallel}(q) - D_{\perp}(q))\frac{q_x q_y}{q^2} = D^{yx}(q), \tag{26}$$

$$D^{zz}(q) = -D^{xx}(q) - D^{yy}(q). \tag{27}$$

Here $D_{\perp}(q)$ and $D_{\parallel}(q)$ are the eugene values of the dipole tensor, which depends only on the absolute value of the wave vector. The eugene vectors of the dipole tensor in this case are parallel and perpendicular to the wave vector and the eugene values are marked correspondingly to this eugene vectors. We have

$$D_{\parallel}(q) = -a + cq - b_1 q^2, \tag{28}$$

$$D_{\perp}(q) = -a + b_2 q^2, \tag{29}$$

where $a = -C_0^{xx} = -C_0^{yy} \approx 5.517$, $c = 4\pi/\sqrt{3} \approx 7.255$, $b_1 = -B_x^{xx} = -B_y^{yy} \approx 1.316$ and $b_2 = B_y^{xx} = B_x^{yy} \approx 0.263$. Let us find the configurations without magnetic charges inside and on the boundaries of the sample, i.e., when div $\boldsymbol{M} = 0$. Then the functional of the energy takes the form (we assume $M_0 = 1$ for simplicity):

$$E = \frac{3}{2}a\int M_z^2\,\mathrm{d}\boldsymbol{r} + \frac{b_2}{2}\int\left(\left(\frac{\partial M_y}{\partial x_i}\right)^2 + \left(\frac{\partial M_y}{\partial x_i}\right)^2 + \left(\frac{\partial M_z}{\partial x_i}\right)^2\right)\mathrm{d}\boldsymbol{r} + \frac{(b_1 - 2b_2)}{2}\int\left(\frac{\partial M_z}{\partial x_i}\right)^2\mathrm{d}\boldsymbol{r} + \frac{c}{2}\int\frac{(\partial M_z/\partial x_i)(\partial M_z/\partial x_{i1})}{|\boldsymbol{r} - \boldsymbol{r}_1|}\,\mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}_1 - \int HM_z\,\mathrm{d}\boldsymbol{r}, \tag{30}$$

where $\boldsymbol{M}$ is the magnetization. The first term in the expression is similar to the energy of the anisotropy, the second and the third ones are similar to the exchange energy, the fourth and the fifth ones are the magnetostatic energy and the energy of the interaction with the external magnetic field perpendicular to the sample plane. So the problem of equilibrium configuration of the dipole system reduces to the traditional micromagnetic problem. The specific feature of the considered system is a small value and anisotropy of the pseudoexchange terms. Nevertheless, these terms can play a significant role. Using the polar variables (the polar axis perpendicular to the plane of the system), from the conditions of the energy functional minimum one can obtain the system of equations for polar and azimuthal angles (θ and ϕ):

$$-\beta\cos\theta\sin\theta - \alpha\Delta\theta + \alpha\sin\theta\cos\theta(\nabla\phi)^2 + \gamma\sin\theta\cos\theta(\nabla\theta)^2 - \gamma\nabla(\sin^2\theta\nabla\theta) + H\sin\theta - \frac{c\sin\theta}{2}\int\frac{\partial\cos\theta}{\partial x_{i1}}\frac{\partial}{\partial x_{i1}}\frac{1}{|\boldsymbol{r} - \boldsymbol{r}_1|}\,\mathrm{d}\boldsymbol{r}_1 = 0, \tag{31}$$

$$\nabla(\sin^2\theta\nabla\phi) = 0, \tag{32}$$

where $\alpha = b_2$, $\beta = 3a$ and $\gamma = (b_1 - 2b_2)$. Using the polar coordinates in the plane of the system ($x = \rho\cos\varphi$, $y = \rho\sin\varphi$) we find the solution of Eqs. (31) and (32) in a vortex form

$$\theta = \theta(\rho), \quad \phi = \frac{\pi}{2} + \varphi. \tag{33}$$

Eq. (32) is satisfied automatically, from Eq. (31) one obtains:

$$\alpha\Delta\theta + \left(\beta - \frac{\alpha}{\rho^2}\right)\cos\theta\sin\theta + \gamma\left(\sin\theta\cos\theta\left(\frac{\partial\theta}{\partial\rho}\right)^2 + \left(\sin^2\theta\frac{\partial^2\theta}{\partial\rho^2}\right)\right) - H\sin\theta + \frac{c\sin\theta}{2}\int\frac{\partial\cos\theta}{\partial x_i}\frac{\partial}{\partial x_{i1}}\frac{1}{|\boldsymbol{r} - \boldsymbol{r}_1|}\,\mathrm{d}\boldsymbol{r}_1 = 0. \tag{34}$$

Without an external magnetic field there is a solution: $\cos\theta = 0$, which will be referred to as a 'uniform' vortex. It corresponds to the vortex configuration which was found numerically in [16]. Its energy in the long wave approximation tends to infinity due to the divergence of the pseudoexchange energy in the center of the vortex. Really, this divergence is cut off on the scale of the lattice parameter. For Eq. (34) the solutions which do not have such divergences are known. They are named cone vortices [22,24]. These configurations can be obtained by solving Eq. (34) with the boundary condition $\theta(0) = 0$. The question about a configuration which yields the minimum of the energy for the ordinary exchange magnetics, if $h = 0$, was discussed in Ref. [25]. It was shown that there is a critical ratio of the exchange constant and an easy-plane anisotropy constant, which divides the ground state into the form of an ordinary vortex and one in the form of a cone vortex. As in a pure dipole magnet this ratio is small, the main state in the form of an ordinary vortex is more probable. Let us show that the situation changes in the external magnetic field near the saturation value.

If $h = H/\beta \to 1$, angle $\theta \to 0$. Let us substitute new variables $x = \rho\sqrt{(1-h)/l_0}$, $f = \theta/\sqrt{2(1-h)}$ ($l_0^2 = \alpha/\beta$) in Eq. (34). Neglecting terms of order $\sim\theta^4$ and higher, and also the term θ^3/x^2, one obtains the equation of Gross–Pitaevsky:

$$\frac{\partial^2 f}{\partial x^2} + \frac{1}{x}\frac{\partial f}{\partial x} + \left(1 - \frac{1}{x^2}\right)f - f^3 = 0. \tag{35}$$

Its solution has the form of

$$f \sim x, \quad x \to 0,$$

$$f \approx 1 - 1/2x^2, \quad x \to \infty. \tag{36}$$

So, neglecting of θ^3/x^2 is well confirmed. Let us analyze qualitatively the behavior of a vortex in a disk with a decrease of the external magnetic field. First, it is necessary to take into account the integral term in Eq. (34). Actually, its order $\simeq (1-h)^2$, as the anisotropic pseudoexchange term has the value $\simeq (1-h)^{5/2}$. The solution for the cone vortex in the ferromagnetic disk, taking the magnetostatics into account, was provided by the method of the trial functions in Ref. [26]. The solution essentially depends on the ratio of the sample dimension and the magnetic length. For samples with a small radius the solution is qualitatively similar, in large samples the radial domain structure appears. This situation is more realistic for the considered dipole magnet, as the magnetic length l_0 is small. So, the ground state of an isotropic dipole magnetic of a disk shape is a vortex configuration. The energy of such a state is determined by the highest terms of the long wave approximation. In the first approximation one may limit oneself by the quadratic terms which are similar in form to the ordinary exchange terms of the Geisenberg magnetics. Moreover, these terms define the distribution of the magnetization, if the sample is placed in the magnetic field with a value near the saturation.

With a deviation of the rhombic angle from $\pi/3$, an anisotropy appears in the plane of the system. If the size of the sample is smaller than the longitude connected with this anisotropy, the ground state retains a vortex [22,24]. In the opposite case the magnetic field divides into domains. Their size depends, as usual, on the sample shape and on the domain wall energy which is defined by the discrete character of the system. In the external magnetic field perpendicular to the easy axis (0x), the energy of the domain wall can be written in the long wave approximation as

$$E = \frac{1}{2}\int\!\!\left(C_0^{xx}M_x^2 + C_0^{yy}M_y^2 + B_y^{xx}\left(\frac{\partial M_x}{\partial y}\right)^2 - B_y^{yy}\left(\frac{\partial M_y}{\partial y}\right)^2\right)\mathrm{d}y + \frac{2\pi}{ab} \times \int_{r\neq r_1} \frac{(\partial M_y(y)/\partial y)(\partial M_y(y_1)/\partial y_1)}{|\boldsymbol{r}-\boldsymbol{r}_1|}\,\mathrm{d}x\,\mathrm{d}x_1\,\mathrm{d}y\,\mathrm{d}y_1 - H\int M_y\,\mathrm{d}y, \tag{37}$$

where M is the magnetization. The expressions for C_0^{xx}, C_0^{yy}, B_y^{xx} and B_y^{yy} are similar to that of $D_{zz}(q)$.

If the external magnetic field is near the saturation value $H \to M_0\,(C_0^{yy} - C_0^{xx})$, then the difference between the directions of the magnetization in the neighboring domains is small. Using the angle variables θ and limiting our consideration to the terms $\sim\theta^3$, for the equilibrium angle one obtains the equation

$$l\frac{\partial^2\theta}{\partial y^2} + 2\theta(\theta_\infty^2 - \theta^2) = 0, \tag{38}$$

where $\theta_\infty = \theta(\infty) = \sqrt{2(1-h)}$, $h = H/(C_0^{yy} - C_0^{xx})$. It has the solution

$$\theta = \theta_\infty \mathrm{th}\left(\frac{y}{l}\right), \quad l = \sqrt{\frac{4B_y^{xx}}{(C_0^{yy} - C_0^{xx})\theta_\infty^2}}. \tag{39}$$

So, if the value of the external field is near the saturation, the domain wall width l is much greater than the lattice parameter and the long wave approximation is valid. Let us note that the finite width of the domain wall and its energy are caused by the pseudoexchange interaction ($B_y^{xx} \neq 0$). With a decrease of the external field the integer term proportional to $(1-h)^2$ begins to play a role at first. The order of the second pseudoexchange interaction term in Eq. (37) is $(1-h)^{5/2}$. The solution of the problem of the Néel wall is obtained in Ref. [27] (see also Ref. [28]) by the method of the trial functions. It is shown that the integral magnetostatic term gives rise to the power behavior of θ at long distances. Nevertheless, the core of the wall narrows. Without the external magnetic field the domain wall becomes very sharp and the long wave approximation is not valid. The right form for the energy of the domain wall in that case is

$$E = \frac{1}{2}\sum_m (C'_x M_x^2(m) + C'_y M_y^2(m)) + \frac{B'_x}{4}\sum_m M_x(m)(M_x(m+1) + M_x(m-1)) + \frac{B'_y}{4}\sum_m M_y(m)(M_y(m+1) + M_y(m-1)) - \frac{4}{ab^2}\sum_m \sum_{n\neq 0} \frac{M_y(m) + M_y(m+n)}{n^2}, \quad (40)$$

where

$$C_x = -\frac{4\zeta(3)}{a^3}, \quad B_x = -\frac{32\pi^2}{a^2\sqrt{2ab}}\exp\left(-\pi\frac{b}{a}\right),$$

$$C_y = \frac{2\zeta(3)}{a^3}, \quad B_y = \frac{32\pi^2}{a\sqrt{2ab}}\left(\frac{\pi}{a} + \frac{1}{b}\right)\exp\left(-\pi\frac{b}{a}\right).$$

The energy of the uniform configuration ($M_x(m) = M_0$, $M_y(m) = 0$) is $E = (B'_x + C'_x)M_0^2/2$. By immediate summation the energy of the dipole system with an infinitely thin wall

$$M_x(m) = M_0, \quad m < 0,$$

$$M_x(m) = -M_0, \quad m \geqslant 0,$$

$$M_y(m) = 0,$$

can be obtained. Evidently, the energy of the infinitely thin domain wall is caused by the pseudoexchange interaction:

$$E = -B'_x M_0^2, \quad B'_x < 0. \quad (41)$$

So, a pseudoexchange interaction will determine the size of the domains in a dipole systems with the FM type of LRO. It should be noted that, due to a small thickness of the domain wall, its movement in the external magnetic field is hindered by pinning on the lattice. The energy of the pinning in our case can be estimated from above by calculating the energy of the intermediate structure appearing while the domain wall is moving from one site of the lattice to the other:

$$M_x(m) = M_0, \quad M_y(m) = 0, \quad m < 0,$$

$$M_x(0) = 0, \quad M_y(m) = M_0, \quad m = 0,$$

$$M_x(m) = -M_0, \quad M_y(m) = 0, \quad m > 0.$$

As the energy of the intermediate configuration is $E = C'_y M_0^2/2$, the field of the pinning which must be exceeded to move the wall is

$$H_{pin} \leqslant (B'_x - C'_x + C'_y)M_0/2. \quad (42)$$

Note that the field of the pinning is less than the saturation field, and the sample is being magnetized by the domain wall movement.

5. Conclusions

In the present work the metastable and nonuniform states of a system of the dipoles with the FM type of LRO on a 2D rhombic lattice are investigated. It is shown that the state with the FM type of LRO is stable for any rhombic angle except $\varphi = 90^\circ$. If $\varphi < \varphi_c$ ($\simeq 76^\circ$), it is the ground state, if $\varphi_c < \varphi < 90^\circ$, the metastable one. Let us note that with $\varphi < \varphi_c$ there can exist a metastable state with the AFM type of LRO. The problem on stability of this state can be investigated as has been done above. The analysis of the stability of the state with the FM type of LRO in an external magnetic field allows to find out the mechanism of the process of the magnetization reversal of the system. With larger rhombic angles it takes place by coherent rotation of the dipoles in the plane of the system, if the angles are small – by antiferromagnetic rotation in the plane perpendicular to the sample.

The nonuniform states of the dipole system, which must exist on bounded lattices were investigated in the long wave limit. The zero term of the energy expansion in terms of the wave vector defines the anisotropy of the system, the linear term corresponds to the magnetostatic interaction of the dipoles in the continuous approximation, the quadratic term is the pseudoexchange interaction, caused by the discrete character of the lattice. It is shown that the energy of the nonuniform configurations of the system depends on a pseudoexchange interaction. In the specific cases (for example, in the external magnetic field, perpendicular to the easy axis) the quadratic term of the expansion largely affects the dipole configuration. For example, appearance of the state with a cone vortex on triangular lattice and the finite width of the domain wall in the external field in anisotropic systems are caused by the pseudoexchange interaction.

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