Hysteresis model with dipole interaction: Devil's staircase like shape of the magnetization curve

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(Received 11 May 2001; revised manuscript received 5 February 2002; published 10 May 2002)

Magnetic properties of two-dimensional (2D) systems of magnetic nanoobjects (2D regular lattices of magnetic nanoparticles or magnetic nanostripes) are considered. Analytical calculations of the hysteresis curve of a system with interaction between nanoobjects are provided. It is shown that during a magnetization reversal the system undergoes a number of metastable states. The kinetic problem of the magnetization reversal was solved for three models. The following results have been obtained: (1) for a 1D system ($T=0$) with a long-range interaction with the energy proportional to r^{-p}, a staircaselike shape of the magnetization curve has a self-similar character (complete ''devil's staircase''). The nature of the steps is determined by the interplay of the interparticle interaction and the coercivity of a single nanoparticle; (2) The influence of thermal fluctuations on the kinetic process was examined in the framework of the nearest-neighbor interaction model. The thermal fluctuations lead to additional splitting of the steps on the magnetization curve. The influence of the coercivity dispersion in the system is also discussed. A simple method to experimentally distinguish the influence of interaction from the coercivity dispersion on the magnetization curve is proposed.

DOI: 10.1103/PhysRevB.65.184433 PACS number(s): 75.60.Jk, 75.75.+a

I. INTRODUCTION

The properties of magnetic nanoobjects and their systems are of active research interest currently owing to the advances in the technology for their fabrication and measurements. The possible applications of such systems are magnetic random access memory[1] or high-resolution magnetic-field sensors.[2] Fundamental interest concerns the understanding of the magnetic behavior of both individual nanoparticles and of arrays of interacting particles. This understanding is necessary to develop any system suitable for technological applications. Many of the magnetic nanosystems under experimental investigation belong to the group of easy axis systems.

What is the main common feature of such magnetic systems from the theoretical point of view? First, they all consist of magnetic coercive objects: systems of magnetic nanoparticles with a perpendicular single-particle anisotropy;[3] chains of magnetic nanoparticles, which have the effective anisotropy axis along the chain due to interparticle dipole-dipole interaction;[4] magnetic nanostripes having a form anisotropy.[5] Second, the process of magnetization reversal in these systems has a thermoactivated nature. For example, the magnetization process in an individual magnetic nanostripe proceeds by nucleation-propagation mechanism.[6] The propagation of the nucleus is very fast. Numerical simulations demonstrate that in a one-dimensional (1D) chain of magnetic nanoparticles the magnetization reversal proceeds through nucleation and followed by propagation of the domain wall.[7] Both a magnetic nanostripe and a chain of magnetic nanoparticles have two stable states with the magnetization directed along the stripe or the chain. A nanoparticle with the perpendicular anisotropy axis also has two stable states and the magnetization reversal in a particle has a thermoactivated nature.[8] Besides, there is a long-range interaction in the systems of such magnetic nanoobjects, which has a magnetostatic nature. In the case of arrays of magnetic particles with anisotropy perpendicular to the array plane the interaction is of effective antiferromagnetic type. Its energy is

$$E_{ij}=\frac{M(r_i)M(r_j)}{|r_{ij}|^3}. \tag{1}$$

Here $M(r_i)$ is the magnetic moment of a particle, r_{ij} is the interparticle distance. In a system of magnetic nanostripes the magnetostatic interaction is also long range. It is caused by magnetic charges appearing on the edges of a stripe in the magnetized state. The dependence of the interaction energy on the interstripe distance is defined as

$$E_{stripe}=2\frac{M(r_i)M(r_j)}{L^2}\left(\frac{1}{r_{ij}}-\frac{1}{\sqrt{r_{ij}^2+L^2}}\right), \tag{2}$$

where $M(r_i)$ is the magnetic moment of a stripe, $r_{i,j}$ is the interstripe distance, and L is the length of the stripes. E is proportional to r^{-1} for the neighboring stripes, $E\sim r^{-3}$ for long distances.

Another system of those mentioned above is a 2D rectangular lattice of magnetic nanoparticles with a single-particle anisotropy of the ''easy-plane'' type. In this case the particles form chains lying along the short side of an elementary rectangle. Due to anisotropy of the dipole interaction, the magnetization of a chain is directed along the chain. The energy of the interchain interaction includes the part (2) connected with the existence of magnetic charges on the chain edges, and the other part caused by a discrete nature of the chain.[9,10]

$$E_{discr}=\frac{8\pi^2 M(r_i)M(r_j)/La^2}{\sqrt{r_{ij}/a}}\exp(-2\pi r_{ij}/a). \tag{3}$$

Here $M(r_i)$ is the magnetic moment of a chain, a is the interparticle (within a chain) distance, r_{ij} is the distance between the chains, and L is their length. The relation E_{discr}/E_{stripe} is proportional to the chain length. So for

rather long chains the nearest-neighbor interaction plays a leading role.[9] The type of the interaction is antiferromagnetic too.

Some attempts were made to solve the problem of the magnetization process in a coercive system with interaction by the numerical methods.[11–13] It was found that the magnetization curves look like a staircase with the steps of different widths. First experimental observation of such steps is reported in Ref. 11.

In our work we present an analytical investigation of the problem of magnetization process in a system of coercive easy-axis nanomagnets. In Sec. II, we examine a 1D model of a system of 2^N magnetic moments with coercivity and long-range interaction decaying at the rate of $1/r^p$. This model corresponds to the system of magnetic nanostripes. We use the cyclic boundary conditions. It is shown that the magnetization curve consists of a series of steps corresponding to formation of superstructures. In the case $N\to\infty$ this staircaselike curve becomes self-similar (complete ''devil's staircase''). The method used for the solution can be easily generalized for the case of a 2D square lattice of the magnetic nanoparticles with a perpendicular single-particle anisotropy. In Sec. III, we solve the problem for a 1D system with the nearest-neighbor antiferromagnetic interaction and single object coercivity at finite temperature. The influence of thermal fluctuations on the magnetization process described in the preceding section is also discussed there. It is shown that defects arising in the superstructures lead to splitting of the steps on the magnetization curve. The influence of the coercivity dispersion is also discussed. A very easy method to estimate the contribution of the interaction and coercivity dispersion in the magnetic properties of the system is proposed.

II. THE MODEL WITH A LONG-RANGE INTERACTION: DEVIL'S STAIRCASE

Let us consider a 1D system of long-range interacting coercive magnetic moments. This model is appropriate for an array of finite length magnetic nanostripes or chains of magnetic nanoparticles. There is an effective antiferromagnetic interaction of a magnetostatic nature between the stripes. We consider its energy in the dimensionless form,

$$\epsilon=\frac{I}{|k-n|^p}\sigma_k\sigma_n, \tag{4}$$

where $\sigma_k=\pm 1$ are the interacting magnetic moments, n and k are the numbers of the magnetic-moment positions, and I is the dimensionless constant of the effective antiferromagnetic interaction ($I>0$). The nearest-neighbor distance is equal to 1. Let the system be originally magnetized so that all $\sigma_k=-1$. The coercivity is considered to be the same for every magnetic moment in the system. The magnetization reversal in a totally magnetized system begins when the field at the place of some magnetic moment exceeds the coercivity of that moment. This critical value is

$$H_1=H_c-2I\xi(p), \quad \xi(p)=\sum_{k=1}^{N}\frac{1}{k^p}, \tag{5}$$

in the case of the infinite chain. Here H_c is the coercivity and it is the same for all magnetic moments. The second term is the field originated at the place of one magnetic moment by all other magnetic moments. In a similar way it can be easily calculated that the reversal ends at the external field value

$$H_2=H_c+2I\xi(p), \tag{6}$$

when the last magnetic moment is reversed. H_2 is greater than H_1 due to the antiferromagnetic interaction in the system. It is interesting to investigate the magnetization curve in the region $H_1<H<H_2$. As the system is one dimensional, its ground state is disordered. Here we will consider the temperatures less than the coercivity ($kT<MH_c$), then the system can have a number of metastable states. There are 2^n (n is the number of the magnetic moments) metastable states at a zero external field if the interaction energy in the system is less than the energy of coercivity.

If the coercivity in a system is small ($kT\gg MH_c$) there are no metastable states in the system, as the fluctuations are higher than the barrier between states. There is only one stable state in this case. It is a ground state. So if we change the external magnetic field the system will consequently pass through a series of ground states. The ground state is totally magnetized (i.e., all magnetic moments are directed along the external field) if the field is less than $-2I\xi(p)$ or larger than $2I\xi(p)$. It is also possible to find the values of the magnetization of the ground states corresponding to the external field in the interval $-2I\xi(p)<H<2I\xi(p)$. This thermodynamic problem was solved in Ref. 14. It was obtained that the magnetization curve has steps and looks like a self-similar devil's staircase in this case.

Here we solve the problem for the case when kT is less than any energy in the system. So the system can be in metastable states and the problem of the magnetization reversal cannot be solved by thermodynamic methods. We have to solve the kinetic problem by a correct choice of the sequence of metastable states the system undergoes when it is magnetized. Such a choice becomes easier if the system consists of an absolutely even number ($N=2^n$) of magnetic moments and has cyclic boundary conditions.

So, magnetization reversal begins when the external field exceeds H_1 (5). Due to fluctuations the place of the first reversed magnetic moment can be chosen arbitrarily. Since interaction here is of the antiferromagnetic type, the external field has to be increased additionally to reverse second magnetic moment. As the interaction decreases with distance, the second reversed magnetic moment must be chosen as far as possible from the first one. It is very easy to choose this place for the system with cyclic boundary conditions [Fig. 1(A)]. In this case the magnetization proceeds through a sequential formation of different superstructures [Fig. 1(B)] that are metastable states. Let us calculate the field values, when some superstructure appears (H_-) and becomes unstable (H_+). At first, we consider only superstructures with periods $m=2^k$, $k=0,1,2,\ldots$ and one reversed magnetic moment per period [such as in Figs. 1(b), 1(c), and 1(e)]. $k=0$ corresponds to the saturated state. The field, when the superstructure with the period equal to m is formed (Fig. 2), is described as

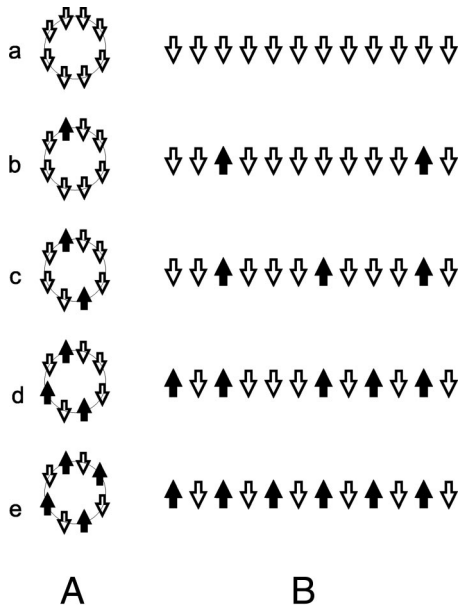

FIG. 1. The reversal process in the cyclic system of eight magnetic moments and the corresponding superstructures in the infinite system. White arrows are for nonreversed magnetic moments; black arrows denote the already reversed ones. The external field increases from (a) to (e). (A) The reversal process in the cyclic system of eight magnetic moments and (B) the corresponding superstructures in the infinite system.

$$H_-(m)=H_c+2I\frac{\xi(p)}{m^p}-\left(2I\xi(p)-2I\frac{\xi(p)}{m^p}\right)$$

$$=H_c-2I\xi(p)+4I\frac{\xi(p)}{m^p}. \tag{7}$$

Here the second term defines the fields of the magnetic moments that have been already reversed. They prevent the chosen magnetic moment from reversal. The term in brackets is the field of other nonreversed magnetic moments. These help

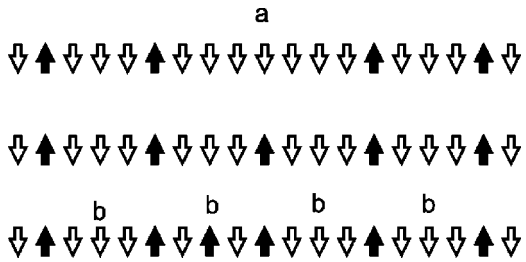

FIG. 2. The process of formation and a following destruction of the superstructure with the period $m=4$. White arrows are for nonreversed magnetic moments; black arrows denote the already reversed ones. H_- is the field when the magnetic moment in the point *a* becomes unstable and the formation of the superstructure takes place. H_+ is the field when the magnetic moments of the points *b* become unstable. Due to fluctuations one of them reverses first and then prevents others from reversal due to long-range antiferromagnetic interaction.

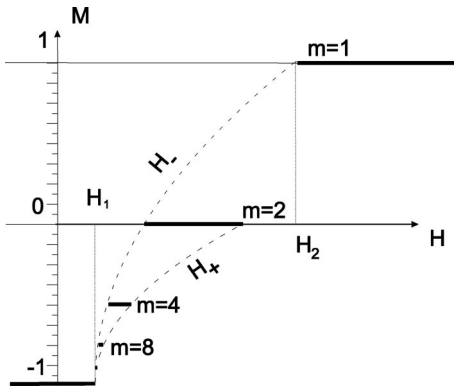

FIG. 3. The steps on the magnetization curve corresponding to simple superstructures ($m=2^k$) with one reversed magnetic moment per period. Magnetization of the corresponding step is $M=(2-m)/m=-1+1/2^{k-1}$.

the chosen magnetic moment to reverse. In a similar way, we can find the field when the superstructure with the period *m* becomes unstable (Fig. 2);

$$H_+(m)=H_c+2I\frac{(2^p-1)\xi(p)}{m^p}$$

$$-\left(2I\xi(p)-2I\frac{(2^p-1)\xi(p)}{m^p}\right)$$

$$=H_c-2I\xi(p)+4I\frac{(2^p-1)\xi(p)}{m^p}. \tag{8}$$

Here we use the relation

$$\sum_{n=1}^{\infty}\frac{1}{(n/2)^p}=\sum_{n=1}^{\infty}\frac{1}{(n+1/2)^p}+\sum_{n=1}^{\infty}\frac{1}{n^p}. \tag{9}$$

Magnetization of the entire system is determined by the superstructure period and is defined as

$$M=\lim_{N\to\infty}\frac{1}{2^N}\sum_{k=1}^{\infty}\sigma_k=\frac{2-m}{m}. \tag{10}$$

So there must be steps on the magnetization curve corresponding to stable superstructures, as the magnetization does not change whereas the magnetic field increases from $H_-(m)$ to $H_+(m)$. Using Eqs. (7),(8), and (10), we can rewrite the dependence of the step edges (Fig. 3) in the form

$$H_-=H_c-2I\xi(p)+4I\xi(p)\left(\frac{M+1}{2}\right)^p, \tag{11}$$

$$H_+=H_c-2I\xi(p)+4I(2^p-1)\xi(p)\left(\frac{M+1}{2}\right)^p. \tag{12}$$

The steps corresponding to the above superstructures do not cover all of the field values between values H_1 and H_2 (Fig. 3). To understand the magnetization behavior of the system in transition from one step to another we must take into account the formation of more complex superstructures

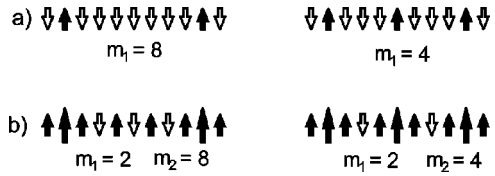

FIG. 4. (a) Simple superstructures (m_1 is a period). White arrows are for nonreversed magnetic moments; black arrows denote the already reversed ones. (b) Complex superstructures characterized by two numbers, m_1 and m_2. Small arrows denote the antiferromagnetic background, big arrows indicate the reversed magnetic moments forming a period of a superstructure.

[Fig. 1(d)]. Let us consider, for example, the magnetization process between $H_+(m=2)$ and H_2 (Fig. 3), that is, how an antiferromagnetic superstructure becomes saturated. The superstructures with periods $m_2=2,4,\ldots,2^l$ $(l=1,2,\ldots)$ appearing with the antiferromagnetic one $(m_1=2)$ as a background are presented in Fig. 4 ($m_1=2, m_2=2$ is the saturated state, $m_1=2, m_2=\infty$ is the antiferromagnetic superstructure). In this case $M=2/m_2$. The expressions for the magnetic fields $H_-(m_2)$ and $H_+(m_2)$ differ from such expressions (12),(11) for simple superstructures. We must take into account the field of the antiferromagnetic background affecting the reversing magnetic moment in this case. This additional field is

$$H=2I\frac{(2^p-1)\xi(p)}{m_1^p}=2I\frac{(2^p-1)\xi(p)}{2^p}. \tag{13}$$

We must take this field into account twice, as formerly it was directed along the external field but now it is directed against it. So we have

$$H_-(m_2)=H_c-2I\xi(p)+4I\xi(p)\left(\frac{M}{2}\right)^p+4I\frac{(2^p-1)\xi(p)}{2^p}, \tag{14}$$

$$H_+(m_2)=H_c-2I\xi(p)+4I(2^p-1)\xi(p)\left(\frac{M}{2}\right)^p + 4I\frac{(2^p-1)\xi(p)}{2^p}. \tag{15}$$

It is obvious that Eqs. (14) and (15) are nearly identical to Eqs. (11) and (12), but now the curves start at the point $M=0$, $H=H_+(m_1=2)=H_c-2I\xi(p)+4I[(2^p-1)\xi(p)/2^p]$, which is the right edge of the step corresponding to the antiferromagnetic structure, instead of $M=-1$, $H=H_1=H_c-2I\xi(p)$ (see Fig. 5). All other steps on the magnetization curve can be obtained in a similar way. Each step corresponding to some superstructure is a base of a series of steps, corresponding to more complex superstructures with the first one as the background. The dependences of H_- and H_+ in all cases are similar, and $H_\pm\sim M^p$. So the picture becomes self-similar (Fig. 5). Complex superstructures are characterized by a series of numbers $m_1,m_2,m_3,\ldots,m_{max}$, where $m_i=2^k$ and $m_i<m_{i+1}$. The maximal number m_{max} is the

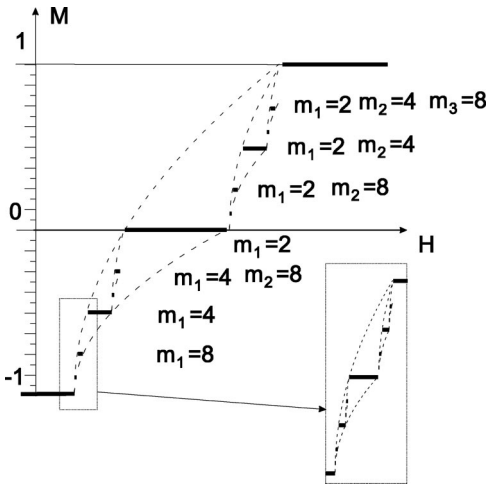

FIG. 5. The self-similar devil's staircase.

period of a superstructure, where $m_1,m_2,\ldots,m_{max-1}$ characterize its background. Narrow steps are between the wider ones. Let us find the total of all steps. A step width can be easily calculated as

$$\Delta H=H_+(m)-H_-(m)=4I\frac{(2^p-2)}{m_{max}^p}\xi(p). \tag{16}$$

The number of steps of the same width depends on m_{max} and $N=m_{max}/2$. The total width is

$$\begin{aligned}\Delta H&=\sum_{n\ \mathrm{even}}^{\infty}N(n)\Delta H(n)\\&=\sum_{n\ \mathrm{even}}^{\infty}\frac{n}{2}4I\frac{(2^p-2)}{n^p}\xi(p)\\&=4I\xi(p)\sum_{k=1}^{\infty}2^{k-1}\frac{(2^p-2)}{2^{kp}}\\&=2I\xi(p)(2^p-2)\sum_{k=1}^{\infty}\left(\frac{1}{2^{p-1}}\right)^k\\&=2I\xi(p)(2^p-2)\frac{(1/2)^{p-1}}{1-(1/2)^{p-1}}=4I\xi(p).\end{aligned} \tag{17}$$

The sum of the widths over all steps is exactly the same as the width of the slope part of the magnetization curve, which is equal to $4I\xi(p)$ according to Eqs.(5) and (6). So the devil's staircase in question is complete as the whole interval of H is ''filled up.''

The difference in the step width is connected with the decaying long-range interaction (4). The wider steps are caused by the interaction of nearer magnetic moments; the narrow ones are caused by the interaction of more distant magnetic moments.

The magnetization values corresponding to the steps are

$$M=\frac{4k-2}{m_{max}}-1, \tag{18}$$

where $k=1,2,\ldots,m_{max}/2$.

It is interesting that the exponent in Eqs. (11) and (12) is equal to the power index in the expression for the interaction (4). So it is possible to find the power index for a long-range interaction in a real system by the experimental measurement of magnetization curves.

There are some reasons for the distortion of the ideal picture considered above. First, it is thermal fluctuations. As $kT>E_{int}(r)$ at long distances, it leads to distortion and disappearance of the narrow steps that are conditioned by the interaction of distant magnetic moments. Besides, thermal fluctuations may give rise to defects in the superstructures. This will be discussed in the following section. The second reason behind the distortion of the ideal magnetization curve is that a real system has bounds. They can play a significant role, as the interaction is long range. Nevertheless, if the dimension of the system is larger than some r_0 $[kT=E_{int}(r_0)]$, the influence of the bounds will be neglected by thermal fluctuations. Finally, the dispersion of the coercivity of different magnetic moments can dramatically change the magnetization curve. Such self-similar behavior can be observed only in the system with small (less than interaction) coercivity dispersion. The method how to distinguish between the influence of the interaction and coercivity dispersion in a possible experiment is discussed in the last section.

In spite of all deficiencies of the proposed model it helps to understand the peculiarities of the magnetization process in the system of coercive magnetic moments with interaction, the nature of the steps on the magnetization curve[11,13,15] and especially the fact that the difference in the step widths is a consequence of the decaying long-range interaction. It also explains the fact of alternation of the narrow and wide steps on the magnetization curve.[11,13] It is very likely that in the general case the magnetization curve has a self-similar character too. The presented model can be easily generalized for the case of a 2D square lattice of interacting magnetic nanoparticles with a perpendicular anisotropy. In this case one must carry out the summation of the dipole sums for the corresponding superstructures on a square lattice. The superstructures must have a square elementary cell in this case, so the dipole sums can be easily calculated.[16]

III. THE NEAREST-NEIGHBOR MODEL: THERMAL FLUCTUATIONS

Let us consider a magnetization process at a finite temperature less than the coercive energy of a single magnetic moment $(kT<H_cM)$ but higher than the interaction energy of widely spaced magnetic moments. In this case the magnetic moments at distances larger than some r_0 $[kT=E(r_0)]$ begin to reverse independently as the energy of their interaction is smaller than the temperature. Nevertheless, the system can be in metastable states as $kT<H_cM$. The kinetics of the magnetization process can in this case be qualitatively described as follows. When the external magnetic field exceeds some value (H_1), a thermoactivated reversal of individual magnetic moments begins. But the magnetic moments that were the first to reverse prevent the neighboring ones (lying at distances smaller than r_0) from reversal due to effective antiferromagnetic interaction. However, more widely spaced magnetic moments can reverse, as their interaction energy in this case is smaller than temperature. The magnetization process is of the Poisson type and ends when the distance between the neighboring reversed magnetic moments is in the interval $r_0<r<2r_0$. An additional external field is necessary to overcome the antiferromagnetic interaction and to continue the reversal process. So, thermal fluctuations will lead to distortion of the ideal picture described in the preceding section. It is difficult to take thermal fluctuations into account in the general case. Here we have solved the problem for the situation when the temperature is larger than energy of any interaction in the system except that for the most powerful nearest-neighbor interaction. In this case the problem can be solved in the nearest-neighbor approximation [$p=\infty$ in Eq. (4)]. This model is also appropriate for a planar system of long chains of magnetic nanoparticles when the main term in the interaction is interaction of the nearest neighbors (3). The magnetization of the chains is directed along them due to effective anisotropy of a magnetostatic nature. The form of the hysteresis in this case is shown in Fig. 6. The magnetization reversal starts at the field value $H_1=H_c-2I$, as the antiferromagnetic interaction helps the external field. The magnetic moments begin to reverse the magnetization due to thermal fluctuations. But if one magnetic moment reverses, it prevents the neighboring ones from being reversed as the effective field of the interaction is opposite to the external field. A chaotic pattern of the magnetization reversals may cause formation of defects (Fig. 7). So, it is impossible to reach the antiferromagnetic state with $M=0$ at this value of the external field. An additional external field $(H=H_c)$ is necessary to reverse the defects. Then defects change their sign (Fig. 7).

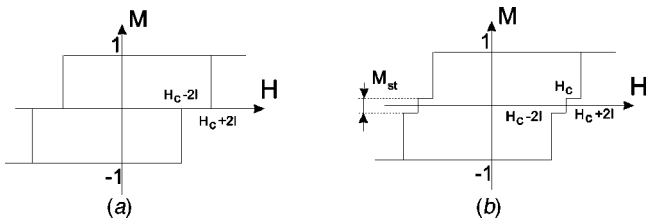

FIG. 6. The hysteresis loop in the case of the nearest-neighbor interaction. (a) At $T=0$ the step corresponds to antiferromagnetically ordered superstructure. (b) The step is split into two steps due to formation of defects by thermal fluctuations. M_{st} is the step height.

The magnetization reversal ends at the field $H_2=H_c$

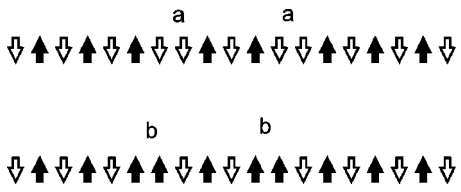

FIG. 7. (a) Defects appearing due to Poisson type of magnetization process and (b) reversed defects. White arrows are for nonreversed magnetic moments; black arrows denote the already reversed ones.

$+2I$ when the system becomes totally magnetized. So, two steps appear on each branch of the hysteresis loop. Their width is $\Delta H=2I$, i.e., it depends on the interaction value. The magnetization value M_{st} corresponding to the step depends on concentration of the defects. It is a special problem to find this concentration. Let us consider the kinetics of the appearance of the defects in a 1D system of N magnetic moments. First, all magnetic moments are magnetized against external magnetic field ($\sigma_k=-1$). When a reversal process begins, the reversed magnetic moments begin to divide the system into regions of yet nonreversed magnetic moments (we will refer to them as ''domains''). In the process of magnetization reversal the number of domains increases and their widths decrease. Let P_n be the number of domains consisting of n nonreversed magnetic moments. Then

$$M=\sum_{n=1}^{N-2} P_n-\sum_{n=1}^{N-2} nP_n. \tag{19}$$

Let α be the probability of reversal of some magnetic moment per unit time. As all magnetic moments have the same coercivity, α is independent of the position of a magnetic moment and equal for all the magnetic moments that do not have reversed neighbors. For all the magnetic moments that have reversed neighbors $\alpha=0$. As the interaction is of the nearest-neighbor type, α does not depend on a configuration of the system. The value of α depends on micromagnetic properties of objects represented by the magnetic moments in our model. Nevertheless, this value does not affect the final result (29). So

$$\frac{\partial P_n}{\partial t}=-\alpha(n-2)P_n+2\alpha\sum_{k=n+2}^{N} P_k, \quad n>2. \tag{20}$$

The first term defines a decrease of the number of domains due to their division into smaller ones; the second term describes the appearance of new domains due to division of the wider ones. As domains consisting of one or two nonreversed magnetic moments cannot be divided further, they only increase in number. So

$$\frac{\partial P_n}{\partial t}=2\alpha\sum_{k=n+2}^{N} P_k, \quad n=1,2. \tag{21}$$

It is obvious that α depends on temperature, but as we are seeking for the states stable at $t\to\infty$, α does not affect the final result. It can be easily checked, that

$$\frac{\partial}{\partial t}\sum_{n=1}^{N}(n+1)P_n=0, \tag{22}$$

i.e., the whole number of magnetic moments in the system is constant. In the course of time ($\alpha t\gg 1$) only the domains with $n=1,2$ remain. So $P_n(t\to\infty)=0$ for $n>2$ and the magnetization value corresponding to the step (Fig. 6) is proportional to the concentration of the defects. So $M=-P_2(t\to\infty)$. We now use the Laplace transformation and define

$$p_n(s)=\int_0^\infty P_n(t)e^{-st}dt. \tag{23}$$

Then, according to (20),

$$p_n(s)=\frac{2\alpha}{\alpha(n-2)+s}Q_n(s), \tag{24}$$

where

$$Q_n(s)=\sum_{k=n+2}^{N} p_k(s). \tag{25}$$

For $s\to 0$

$$Q_n(0)=\sum_{k=n+2}^{N} p_k(0)=\sum_{k=n+2}^{N}\frac{2}{k-2}Q_k(0). \tag{26}$$

In the recursive form this equation can be written as

$$Q_n(0)=Q_{n+1}(0)+\frac{2}{n}Q_{n+2}(0). \tag{27}$$

To find the magnetization value corresponding to the step it is necessary to calculate $P_2(t\to\infty)$. It is obvious that

$$P_2(t\to\infty)=\lim_{s\to 0} sp_2(s)=2\alpha Q_2(0). \tag{28}$$

If the maximum number of the magnetic moments in the system is N, $Q_2(0)$ can be found from Eq. (27) with the initial conditions $Q_{N-2}=Q_{N-3}=p_N(0)$, which are the sequence of Eq. (25). In its turn $p_N(0)=1/\alpha(N-2)$, according to Eq. (20). The solution of the recursive equation was found numerically as

$$M_{st}=2\alpha\lim_{N\to\infty}\frac{Q_2(0,N)}{N}\approx 0.134. \tag{29}$$

As $Q_2(0,N)$ is proportional to α^{-1}, the result does not depend on the value of α. So the formation of defects during a magnetization process leads to appearance of two steps (instead of one, Fig. 6) on the magnetization curve in the case of the nearest-neighbor interaction. Due to fluctuating nature of the magnetization process the antiferromagnetic ground state (if $H=0$) cannot be achieved. One may expect that in the case of long-range interaction thermal fluctuations will lead to similar splitting of steps on the magnetization curve.

IV. DISCUSSION

By means of simple models we have investigated the magnetization processes in the systems of coercive magnetic objects with interaction. The reason behind the formation of steps on the magnetization curve is investigated. It is shown that the magnetization curve can have a self-similar nature. Its form is calculated in the case of long-range interaction with $E\sim 1/r^p$. The suggested model explains the nature of the steps and especially the fact of the alternation of wide and narrow steps on magnetization curves observed both experimentally[11] and by numerical simulations.[13,17] The in-

fluence of the thermal fluctuations is analyzed in the framework of the nearest-neighbor approximation. It is shown that fluctuations lead to splitting of the steps on the magnetization curve.

Let us estimate values of fields of a magnetostatic interaction and possible widths of steps on magnetization curves in experimentally studied magnetic nanosystems, such as those represented in Refs. 3,4 and 5. The best system to investigate multiple steps is rectangular lattices of magnetic nanoparticles (pillars) with a perpendicular anisotropy. For example, for usual parameters of this systems (Ni pillars 110 nm in diameter, 240 nm in height, and 190 nm of lattice period) the nearest-neighbor interaction according to Eq. (1) will be approximately 200 Oe. So the widths of the main and second step can be estimated by Eq. (16) as 600 Oe and 75 Oe, respectively. Actually these will be less due to discussed effect of splitting. This splitting can also be observed for rectangular lattices of the magnetic nanoparticles with in-plane anisotropy. For the particles with dimensions equal to interparticle spacing, formula (3) gets the following estimation for a field of a magnetostatic interaction between chains of the particles:

$$H=\frac{\pi^2 M}{\sqrt{\beta}}[\exp(-2\pi)]^{\beta n}. \tag{30}$$

Here β is aspect ratio, M is magnetization of particle, and n is the number of neighbors. So for usual materials with a magnetization of 1000 Gs and small aspect ratios of a lattice, H_{int} is approximately 20 Oe for the nearest-neighbor chains and 0.04 Oe for the second neighbor. Such system can be considered as a system with nearest-neighbor interaction. The width of two splitting steps can be estimated as 40 Oe.

But besides long-range interaction there can be another reason for the steps on a magnetization curve. Namely, the dispersion of the coercivity of different magnetic moments in the system (even without any interaction) leads to appearance of steps on magnetization curve. How can one distinguish between these effects? As a matter of fact, understanding of their difference is very important to interpret experimental data.

To understand the situation we have considered the simplest model of interacting magnetic moments with dispersion of their coercivity in the mean-field approximation. Within this model the interaction is independent of distance and $\epsilon = I/N$. It should be mentioned that this model is appropriate in the case of very long stripes when $L > Na$ (L is the stripe length, a is the interstripe distance, and N is the whole number of stripes). Let us examine a hysteresis loop for the system without coercivity dispersion. The dependence of the magnetization on an external field is linear in this case, as

$$H=H_c+JM. \tag{31}$$

H_c is the coercivity, JM is the field of interaction in the mean-field approximation. There is a slope of the branches of the magnetization curve [Fig. 8(a)] instead of steps in the case of an infinite system. The main feature of a magnetization process in a coercive system with antiferromagnetic interaction is connected with multistability of the system. It means that, if we change the sign of the changing external field, the system does not change its magnetization immediately. First, it transits through the whole hysteresis loop from one branch to the other [from point A to point B, Fig. 8(a)], and then begins to change its magnetization according to the new branch. In this case $|H_A - H_B| = 2H_c$. It should be noted that such multistable behavior does not depend on the interaction manner. It is the same in the case of long-range decaying interaction (Sec. II) or the nearest-neighbor one (Sec. III). The same multistability was obtained by numerical simulation.[17] So different states of the system (characterized by the different magnetizations) correspond to the same value of the external field. We will refer to such multistability as interaction-type (I-type) multistability, because multistability can exist in a system of noninteracting magnetic moments with different values of coercivity also. In this case magnetization reversal begins when the field reaches the value $H_1 = H_{Cmin}$, when the reversal of the magnetic moments with the smallest coercivity starts. The reversal process is finished at the field value $H_2 = H_{Cmax}$, corresponding to the largest coercivity in the system. The hysteresis loop in this case is similar to the one for the system with interaction (as its branches can have a similar slope in the case of $\Delta H_c = 2I$ and a uniform distribution of H_c), but the transitions inside the loop differ. If we change the direction of the reversal process in point A (Fig. 8) in this case, the magnetization does not change until the external field reaches the value of $-H_1$ when the reversal of the magnetic moments with the smallest coercivity happens. We will refer to such multistability as coercivity-type (C-type). The hysteresis

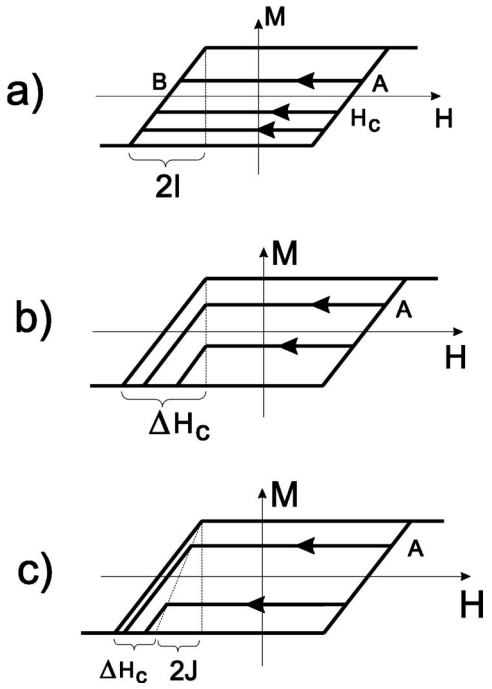

FIG. 8. The hysteresis curve in the mean field approximation. (a) for the system with interaction with the same coercivity of any magnetic moment; (b) for the coercivity dispersion (uniform distribution of the coercivity) and without interaction; and (c) for the system with both interaction and coercivity dispersion.

loop of the system with both interaction and coercivity dispersion can be easily calculated too. It is presented in Fig. 8(c). So one can distinguish between the interaction and the coercivity dispersion by the behavior of the magnetization within the hysteresis loop and by analysis of the multistability type of the system. We believe that a self-similar behavior of the magnetization can be experimentally observed in the systems with a small dispersion of coercivity, that is, in systems that demonstrate the *I*-type of multistability. Note that in the experimental works we know on the type of the multistability, this behavior is not examined in spite of the simplicity, on one hand, and significance, on the other hand, of such investigation.

ACKNOWLEDGMENTS

We are grateful to Professor A. A. Andronov for helpful discussions. The work was supported by the Russian Foundation for Fundamental Research (N 00-02-16485).

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